Generating Uniformly Distributed Pareto Optimal Points for Constrained and Unconstrained Multicriteria Optimization

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Abstract

This paper proposes a novel approach to generate a uniform distribution of the optimal solutions along the Pareto frontier. We make use of a standard mathematical technique for optimization namely line search and adapt it so that it will be able to generate a set of solutions uniform distributed along the Pareto front. To validate the method, numerical bi-criteria examples are considered. The method deals with both unconstrained as well as constrained multicriteria optimization problems.

1 Introduction

Multiobjective Optimization Problems (MOP's) are ones of the hardest optimization problems, which arise in many real-world applications. Due to the increasing interest in multiobjective optimization, several algorithms for dealing with such problems have been proposed recently [1] [2][3][6][7][8]. Most of these algorithms use an iterative method to generate multiple points approximating the Pareto set. A Pareto optimal set is regarded as the mathematical solution to a multicriteria problem. In continuous problems, the number of Pareto optimal solutions is usually infinite. Only in relatively simple cases the entire Pareto optimal set can be determined analytically. In a typical problem, we must be satisfied by obtaining enough Pareto optima to cover the minimal set in the criteria space properly. This computed subset of Pareto optima could be called as a representative Pareto optimal set and its quality can be judged for example by its ability to cover the whole minimal set evenly. In some problems, however, the cost of generating just one Pareto optimum may become so high that the designer can afford only a few Pareto optimal solutions. Before performing numerical optimization a suitable generation strategy is to be selected, which guarantees that only Pareto optima are obtained.

The paper deals with the generation of uniformly distributed Pareto points. A scalarization of the objectives is used in order to transform the multiobjective optimization problem into a single objective optimization problem. A line search based technique is used to obtain an efficient solution. Starting with this solution, a set of efficient points are further generated, which are widely distributed along the Pareto frontier using again a line search based method but involving Pareto dominance relationship.

Rest of the paper is organized as follows. The multiobjective algorithm used is presented in Section 2. The numerical examples used to emphasize the efficiency of the used approach are presented in Section 3 followed by Conclusions in Section 4.

2 The Multicriteria Approach

The line search [3] is a well established optimization technique. The standard line search technique is modified so that it is able to generate the set of non-dominated solutions for a multiobjective optimization problem (MOP) [4]. The approach comprises of two phases: first, the problem is transformed into a SOP and a solution is found using a line search based approach. This is called as convergence phase. Second, a set of Pareto solutions are generated starting with the solution obtained at the end of convergence phase. This is called as spreading phase.

Consider the MOP formulated as follows:

Let $\Re^m$ and $\Re^n$ be Euclidean vector spaces referred to as the decision space and the objective space. Let $X \subset \Re^m$ be a feasible set and let f be a vector-valued

INFOS2008, March 27-29, 2008 Cairo-Egypt
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objective function f: $\Re^m \rightarrow \Re^n$ composed of n real-valued objective functions f=(f_1, f_2,..., f_n), where f_k: $\Re^m \rightarrow \Re$, for k=1,2,..., n. A MOP is given by:

$$\min\ (f_1(x),\ f_2(x),\ldots,\ f_n(x)),$$

$$\text{subject to } x \in X.$$

The MOP is further transformed into a single objective optimization problem (SOP) by considering:

$\min\ F = \sum_{i=1}^{n} f_i^3(x)$
subject to $x \in X$.

Direction and step setting

The direction is set as being a random number between 0 and 1.

The step is set as follows:

α_k=2+$\frac{3}{2^{2k}+1}$ (1)
where k refers to the iteration number.

Convergence phase

Initially, a set of points are randomly generated within the given boundaries. The line search with direction and step set as mentioned above is applied for a given number of iterations for each of these points. Afterwards, the boundaries of the definition domain are modified taking into account the partial derivatives as follows:

Let x be the best result obtained in the previous set of iterations:

For each dimension $i$of the point x, the first partial derivative with respect to this dimension is calculated. This means the gradient of the objective function is calculated which is denoted by g. Taking this into account, the bounds of the definition domain for each dimension are re-calculated as follows:

if $g_i = \frac{\partial F}{\partial x_i} < 0$then *lower bound* $=x_i$
if $g_i = \frac{\partial F}{\partial x_i} > 0$then *upper bound* $=x_{i;}$

The search process is re-started by re-initializing a new arbitrary point between the newly obtained boundaries.

Spreading phase

At the end of the convergence phase, a solution is obtained. This solution is considered as an efficient (or Pareto) solution. During this phase and taking into account of the existing solution, more efficient solutions are to be generated so as to have a thorough distribution of all several good solutions along the Pareto frontier. In this respect, the line search technique is made use of to generate one solution at the end of each set of iterations. This procedure is applied several times in order to obtain a larger set of non-dominated solutions. The following steps are repeated in order to obtain one non-dominated solution:

Step 1. A set of nondominated solutions found so far is archived. Let us denote it by *NonS*. Initially, this set will have the size one and will only contain the solution obtained at the end of *convergence phase*.

Step2. We apply line search for one solution and one dimension of this solution at one time. For this:

Step 2.1. A random number i between one and |NonS| ($|\cdot|$ denotes the cardinal) is generated. Denote the corresponding solution by $nonS_i$.

Step 2.2. A random number j between one and the number of dimensions (the number of decision variables) is generated. Denote this by $nonS_{ij}$.

Step 3. Line search is applied for $nonS_{ij}$.

Step 3.1. Set p=random.

Step 3.2. Set α (which depends on the problem, on the number of total nondominated solutions which are to be generated, etc.).

Step 3.3. The new obtained solution *new_sol* is identical to $nonS_i$ in all dimensions except dimension j which is:

$new\_sol_j = nonS_{ij} + \alpha \cdot p$

Step 3.4. if ($new\_sol_j >$ *upper bound*) or ($new\_sol_j <$ *lower bound*)
then new_sol_j = *lower bound* + random · (*upper bound* – *lower bound*).

Step 4. if $F(new\_sol) > F(nonS_1)$
then discard *new_sol*
else if *new_sol* is nondominated with respect to the set *NonS*
then add *new_sol* to *NonS* and increase the size on *NonS* by 1.

Go to step 2.

Step 5. Stop

3 Experiment Results

In order to emphasize the performance of the above technique to generate uniformly distributed Pareto points we make use of two multiobjective optimization test problems: and unconstrained MOP and a constrained MOP.

3.1 Unconstrained example (Problem 1)

Consider the following problem [9]:

Min f={$f_1(x)$, $f_2(x)$}
$f_1(x)$=cosh(x)
$f_2(x) = x^2$-12x+35

The multiobjective approach used the following parameters:

- number of re-starts: 10;
- number of iteration per each re-start: 10;
- α for the spreading phase (its value depends on the number of Pareto solutions which are to be generated).

The Pareto frontier generated using 100 solutions is depicted in Figure 1 and using 1000 solutions in Figure 2. The value of α is 4.9, while the Pareto front consists of 100 points and 9.5 when the Pareto front has 1000 points.

3.2 Constrained example (Problem 2)

Considering the following test function with equalities and inequalities constraints [9]:

Min f={$f_1(x)$, $f_2(x)$}
$f_1(x)$=$(x_1$-2$)^2$+$(x_2$-1$)^2$
$f_2(x) = x_1^2 + (x_2 - 6)^2$

s.t.

$$g_1(x) = x_1^2 - x_2 \le 0$$
$$g_2(x) = 5x_1^2 + x_2 \le 10$$
$$g_3(x) = x_2 \le 5$$
$$g_4(x) = -x_1 \le 0$$

The Pareto frontier generated using 100 solutions is illustrated in Figure 3 and using 1000 solutions in Figure 4. The value of α is 5.4 and the Pareto front consists of 100 points and 7.2 when the Pareto front has 1000 points.

Figure 1. Pareto front obtained with 100 solutions for Problem 1.

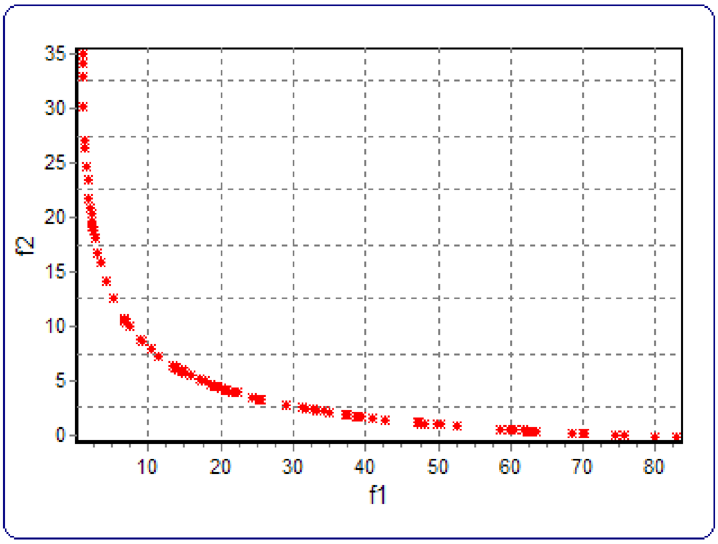


Figure 2. Pareto front obtained with 1000 solutions for Problem 1.

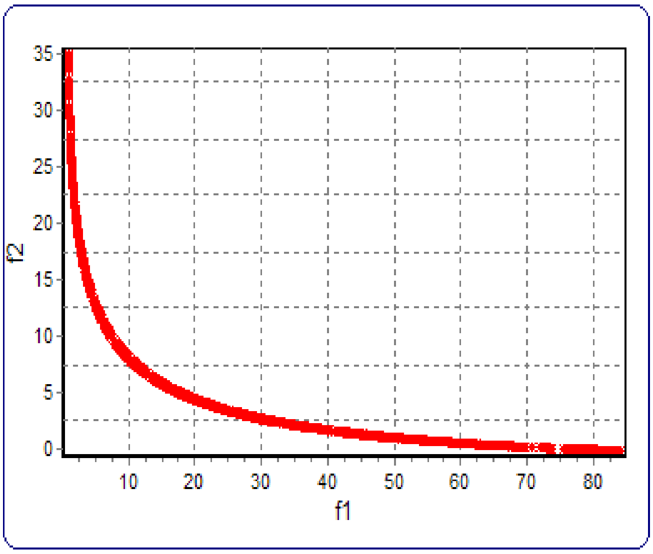


Figure 5. Distributions of solutions along the Pareto frontier for Problem 2 by considering: (a) α=1.5 and (b) α=9.7.

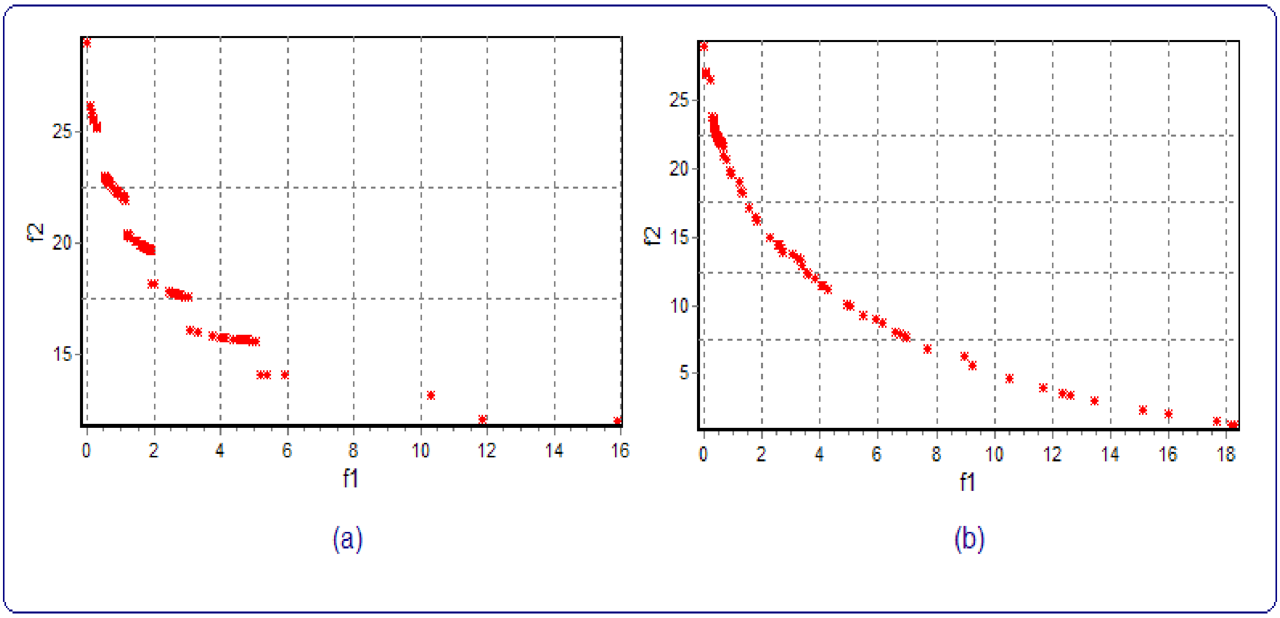


Figure 3. Pareto front obtained with 100 solutions for Problem 2.

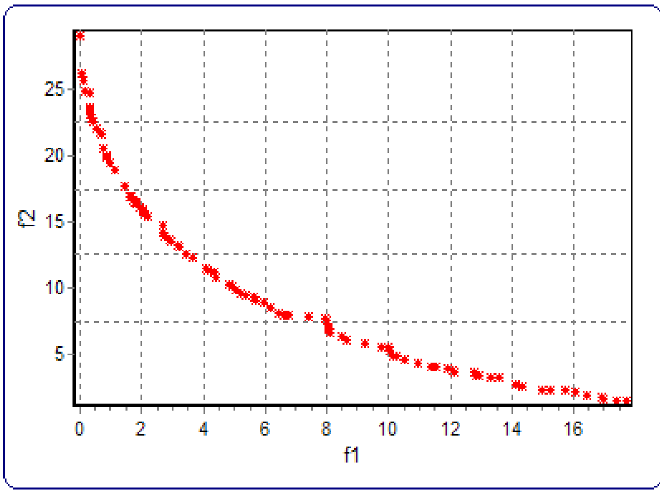
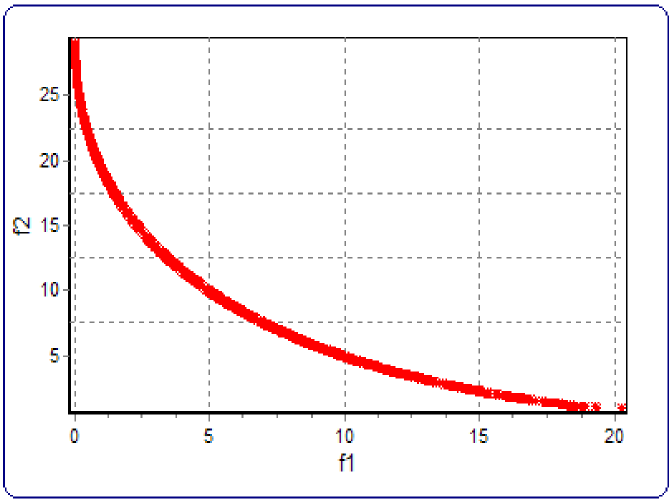


Figure 4. Pareto front generated using 1000 points for Problem 2.



One of the challenges of this approach is to find an adequate value for α. The values obtained in our examples were set based on several experiments. For example, Figure 5 depicts two other distributions of the solutions along the Pareto front which are not uniform obtained for α=1.5 (Figure 5 (a)) and α=9.7 (Figure 5 (b)).

4 Conclusions

The paper proposed a novel method for finding a set of uniformly distributed Pareto solutions along the Pareto frontier. A line search based technique is applied in order to obtain one solution. Starting from this solution, a simplified version of the initial line search is used in order to generate solutions with a well distribution on the Pareto frontier. Numerical experiments performed on unconstrained as well as constrained multiobjective optimization test problems show that the approach presented here is able to converge very fast and provide a very good distribution of solution along the Pareto frontier. The approach uses a parameter whose setting gives a better or a worse distribution. The value of this parameter is set based on some experimental tests but one of our future research plans is to find a better way to deal with setting the right value for this parameter.

References

[1] Das, I. and Dennis, J., A Closer Look at Drawbacks of Minimizing Weighted Sums of Objective for Pareto Set Generation in Multicriteria Optimization Problems, Structural Optimization, 14, pp. 63–69, 1997.

[2] Dumitrescu, D., Grosan, C., and Oltean, M. Evolving Continuous Pareto Regions, Evolutionary Multiobjective Optimization: Theoretical Advances and Applications, Abraham A., Jain L. and Goldberg R. (Eds.), Springer Verlag, London, ISBN 1852337877, 167-199, 2005

[3] Ehrgott, M., and Wiecek, MM., Mutiobjective Programming, In Multiple Criteria Decision Analisys: State of the art surveys, J. Figueira et al. (Eds.), International Series in Operations Research & Management Science, Volume 78, Springer New York, 2005.

[4] Gould, N., An introduction to algorithms for continuous optimization, Oxford University Computing Laboratory Notes, 2006.

[5] Grosan, C. and Abraham, A., Hybrid line search for multiobjective optimization, 2007 International Conference on High Performance Computing and Communications (HPCC-07), Houston, USA, Springer Verlag, Germany, R. Perrott et al. (Eds.): HPCC 2007, LNCS 4782, pp. 62-73, 2007.

[6] Grosan, C., Abraham, A., Nicoara, M. Search Optimization Using Hybrid Particle Sub-Swarms and Evolutionary Algorithms, International Journal of Simulation Systems, Science & Technology, Volume 6, Nos. 10 and 11, pp. 60-79, 2005.

[7] Messac, A. and Mattson, C. A., Generating Well-Distributed Sets of Pareto Points for Engineering Design using Physical Programming, Optimization and Engineering, 3, pp. 431–450, 2002.

[8] Messac, A., Ismail-Yahaya, A., and Mattson, C. A., The Normalized Normal Constraint Method for Generating the Pareto Frontier, Structural and Multidisciplinary Optimization, 25(2), pp. 86–98, 2003.

[9] Zhang, W, H., Gao, T., A min–max method with adaptive weightings for uniformly spaced Pareto optimum points, Computers and Structures 84, pp. 1760–1769, 2006