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Chaotic dynamic characteristics in swarm intelligence

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Abstract

Swarm intelligence (SI) is an innovative distributed intelligent paradigm whereby the collective behaviors of unsophisticated individuals interacting locally with their environment cause coherent functional global patterns to emerge. The intelligence emerges from a chaotic balance between individuality and sociality. The chaotic balances are a characteristic feature of the complex system. This paper investigates the chaotic dynamic characteristics in swarm intelligence. The swarm intelligent model namely the particle swarm (PS) is represented as an iterated function system (IFS). The dynamic trajectory of the particle is sensitive on the parameter values of IFS. The Lyapunov exponent and the correlation dimension are calculated and analyzed numerically for the dynamic system. Our research results illustrate that the performance of the swarm intelligent model depends on the sign of the maximum Lyapunov exponent. The particle swarm with a high maximum Lyapunov exponent usually achieves better performance, especially for multi-modal functions.

Keywords: Swarm intelligence; Chaos; Particle swarm; Iterated function system

1. Introduction

Swarm intelligence (SI) is mainly inspired by social behavior patterns of organisms that live and interact within large groups of unsophisticated autonomous individuals. In particular, it incorporates swarming behaviors observed in flocks of birds, schools of fish, or swarms of bees, colonies of ants, and even human social behavior, from which the intelligence is emerged [1–3]. SI provides a framework to explore distributed problem solving without centralized control or the provision of a global model. The particle swarm model helps to find optimal regions of complex search spaces through interaction of individuals in a population of particles [4]. It has exhibited good performance across a wide range of applications [5–11].

In the swarm dynamic system, the intelligence emerges from a chaotic balance between individuality and sociality. The chaotic balances are a characteristic feature of the complex system. Many studies on swarm intelligence have been presented and even some improved algorithms were proposed based on the chaotic search behavior. For a given energy or cost function, by following chaotic ergodic orbits [12], a chaotic dynamic system may eventually reach the global optimum or its good approximation with high probability. To enhance the performance of particle swarm optimization (one of the swarm intelligent models), Liu et al. [13] proposed hybrid particle swarm optimization algorithm by incorporating chaos. The proposed chaotic particle swarm optimization combined the population-based evolutionary searching ability of particle swarm optimization and chaotic search behavior. Simulation results and comparisons with the standard particle swarm optimization and several other meta-heuristics have shown that the approach could effectively enhance the search efficiency and greatly improve the searching quality. Since chaotic mapping possesses properties of certainty, ergodicity and stochastic property, Jiang and Etorre [14,15] introduced chaos mapping into the particle swarm optimization algorithm for reactive power optimization and short term hydroelectric system scheduling in a deregulated environment. Empirical results demonstrated that the performance of the algorithms

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was improved obviously owing to its fast convergence and high precision.

However, not much work has been reported in the literature on the chaotic characteristics in swarm intelligence. In fact, several other studies in diverse fields indicated the analysis of the chaotic characteristics contributed to the understanding and applications of those complex systems. Chen [16] investigated the chaotic phenomena in macroeconomic systems, and offered an explanation of the multi-periodicity and irregularity in business cycles and of the low-dimensionality of chaotic monetary attractors. The empirical and theoretical results improved monetary control policy and the approaches to forecasting business cycles. Chialvo et al. [17] studied chaotic patterns of activation and action potential characteristics in the cardiac tissues. Their results indicated an apparent link between the mechanism of low dimensional chaos and the occurrence of reflected responses which could lead to more spatially disorganized phenomena. Frank et al. [18] analyzed the chaotic characteristics in the brain dynamics to predict changes of epileptic seizures. Goldberger et al. [19], Freeman [20] and Sarbadhikari and Chakrabarty [21] illustrated that chaos has a great important influence on brain and the evolutionary relationship between species. The investigations of chaotic dynamics in neural networks [22] promoted the development of neural networks and chaotic neural networks [23,24]. The chaotic balances and their characteristic in swarm intelligence has become very importance for its deeper understanding, application development and designing new computational models.

This paper investigates the chaotic dynamic characteristics in swarm intelligence, and analyzes their relationship with the performance of SI. Particle swarm model is investigated as a case study. The swarm intelligent model is represented as an iterated function system (IFS) [25]. We simulate and analyze the dynamic trajectory of the particle based on the IFS. The Lyapunov exponent and the correlation dimension are calculated and analyzed numerically for the dynamic system. The dependence of the parameters is discussed analytically using function optimization experiments.

The rest of the paper is organized as follows. Particle swarm model is presented in Section 2 and the concepts of iterative function system and its sensitivity is illustrated in Section 3. Dynamic chaotic characteristics are depicted and discussed in Section 4 and finally conclusions are made in Section 5.

2. Particle swarm model

A particle swarm model consists of a swarm of particles moving in a d-dimensional search space where the fitness f can be calculated as a certain quality measure. Each particle has a position represented by a position-vector $\vec{x}_i$ (i is the index of the particle), and a velocity represented by a velocity-vector $\vec{v}_i$. Each particle remembers its own best position so far in a vector $\vec{p}_i$, and its jth dimensional value is $p_{i,j}$. The best position from the swarm thus far is then stored in a vector $\vec{p}_g$, and its jth dimensional value is $p_{g,j}$. During the iteration time t, the update of the velocity from the previous velocity is determined by (1). Subsequently, the new position is determined by the sum of the previous position and the new velocity by (2):

$$v_{i,j}(t) = wv_{i,j}(t-1) + c_1 r_1 (p_{i,j}(t-1) - x_{i,j}(t-1)) + c_2 r_2 (p_{g,j}(t-1) - x_{i,j}(t-1)) \quad (1)$$

$$x_{i,j}(t) = x_{i,j}(t-1) + v_{i,j}(t) \quad (2)$$

where r_1 and r_2 are the random numbers, uniformly distributed within the interval [0, 1] for the jth dimension of ith particle. c_1 is a positive constant termed as the coefficient of the self-recognition component; c_2 is a positive constant termed as the coefficient of the social component. The variable w is the inertia factor, for which value is typically setup to vary linearly from 1 to 0 during the iterated processing. From (1), a particle decides where to move next, considering its own experience, which is the memory of its best past position, and the experience of its most successful particle in the swarm. In the particle swarm model, the particle searches the solutions in the problem space within a range $[-s, s]$ (if the range is not symmetrical, it can be translated to the corresponding symmetrical range). In order to guide the particles effectively in the search space, the maximum moving distance during one iteration is clamped in between the maximum velocity $[-v_{\max}, v_{\max}]$ given in (3), and similarly for its moving range given in (4):

$$v_{i,j} = \operatorname{sign}(v_{i,j}) \min(|v_{i,j}|, v_{\max}) \quad (3)$$

$$x_{i,j} = \operatorname{sign}(x_{i,j}) \min(|x_{i,j}|, x_{\max}) \quad (4)$$

The value of $v_{\max}$ is $\rho \times s$, with $0.1 \leq \rho \leq 1.0$ and is usually chosen to be s, i.e. $\rho = 1$. The pseudo-code for particle-search is illustrated in Algorithm 1.

Algorithm 1. Particle swarm model

```
Initialize the size of the particle swarm n, and other parameters.
Initialize the positions and the velocities for all the particles randomly.
While (the end criterion is not met) do
  t = t + 1;
  Calculate the fitness value of each particle;
  p_g(t) = argmin_{i=1}^{n}( f(p_g(t − 1)), f(x_1(t)), f(x_2(t)), ...,
           f(x_i(t)), ..., f(x_n(t)))
  For i = 1 to n
    p_i(t) = argmin_{i=1}^{n}( f(p_i(t − 1))), f(x_i(t));
    For j = 1 to d
      Update the jth dimension value of x_i and v_i
      according to (1), (3), (2), (4);
    Next j
  Next i
End While.
```

3. Iterated function system and its sensitivity

Clerc and Kennedy have stripped the particle swarm model down to a most simple form [26]. If the self-recognition

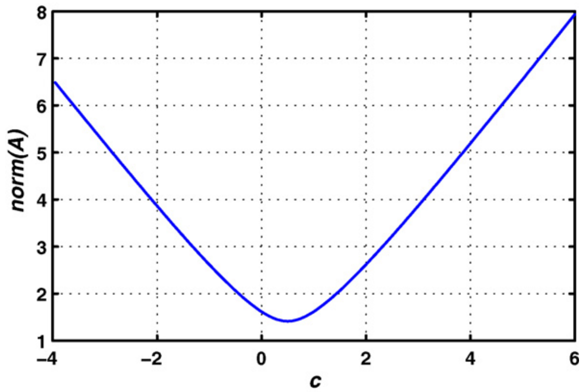

Fig. 1. Norm of A.

component c_1 and the coefficient of the social-recognition component c_2 in the particle swarm model are combined into a single term c, i.e. $c = c_1 + c_2$, the best position $\vec{p}_i$ can be redefined as follows:

$$\vec{p}_i \leftarrow \frac{c_1 \vec{p}_i + c_2 \vec{p}_g}{c_1 + c_2} \tag{5}$$

Then, the update of the particle's velocity is defined by

$$\vec{v}_i(t) = \vec{v}_i(t-1) + c(\vec{p}_i - \vec{x}_i(t-1)) \tag{6}$$

The system can be simplified even further by using $\vec{y}_i(t-1)$ instead of $\vec{p}_i - \vec{x}_i(t-1)$. Thus, the reduced system is then

$$\vec{v}(t) = \vec{v}(t-1) + c\vec{y}(t-1),$$

$$\vec{y}(t) = -\vec{v}(t-1) + (1-c)\vec{y}(t-1)$$

This recurrence relation can be written as a matrix-vector product, so that

$$[\,\vec{v}(t) < myslash > \vec{y}(t)\,]$$

$$= [\,1 < myamp > c < myslash > -1 < myamp > 1-c\,]$$

$$\cdot\, [\,\vec{v}(t-1) < myslash > \vec{y}(t-1)\,]$$

Let

$$\vec{P}_t = [\,\vec{v}_t < myslash > \vec{y}_t\,]$$

and

$$A = [\,1 < myamp > c < myslash > -1 < myamp > 1-c\,]$$

we have an iterated function system for the particle swarm model:

$$\vec{P}_t = A \cdot \vec{P}_{t-1} \tag{7}$$

Thus, the system is completely defined by A. Its norm $\|A\|_2$ (also written $\|A\|$) is determined by c. The relationship of A and its dependence on c is illustrated in Fig. 1.

IFS is sensitive to the values of c. It is possible to find different trajectories of the particle for various values of c. Fig. 2 (a) illustrates the system for a torus when $c = 2.9$; Fig. 2(b), a hexagon with spindle sides when $c = 2.99$; Fig. 2(c), a triangle with spindle sides when $c = 2.999$; Fig. 2(d), a simple triangle when $c = 2.9999$. As depicted in Fig. 2, the iteration time step used is 100 for all the cases. Another system sensitivity instance is illustrated in Fig. 3. It is to be noted that Figs. 2 and 3 illustrate only some 2D representations of the iterated process. A comparison between 2D and 3D is illustrated in Fig. 4.

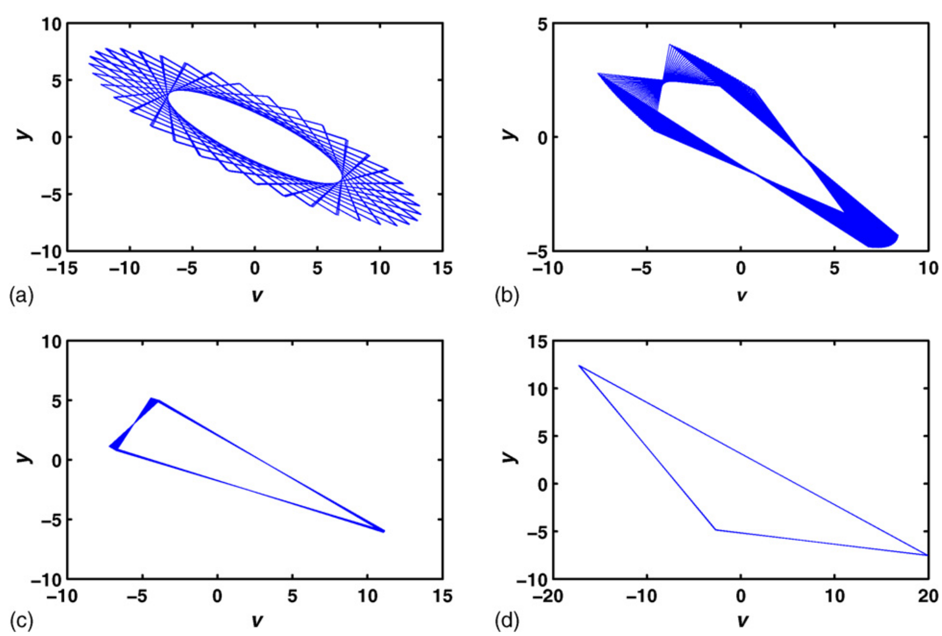

Fig. 2. Trajectory of the particle: (a) $c = 2.9$; (b) $c = 2.999$; (c) $c = 2.999$; (d) $c = 2.9999$.

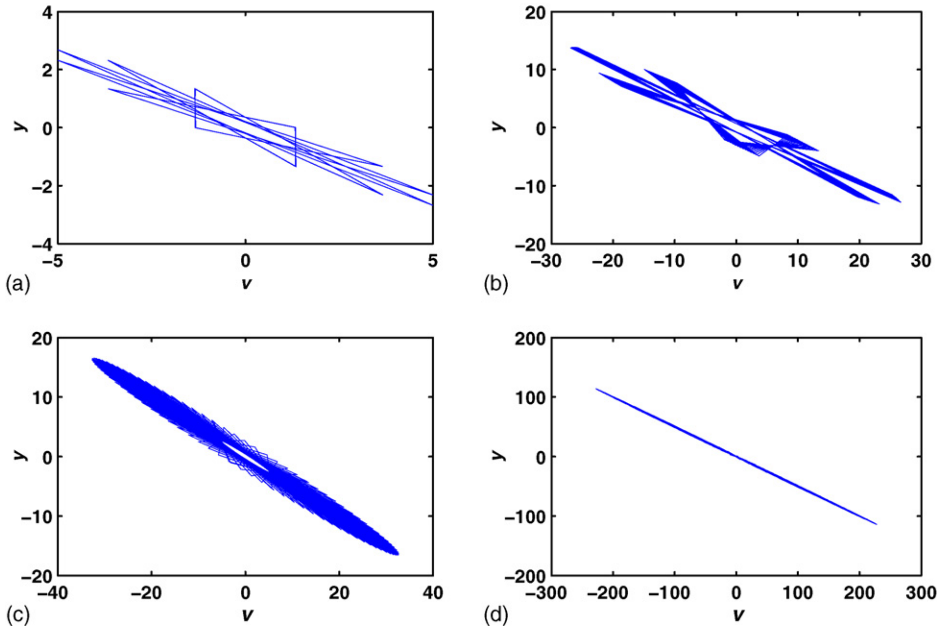

Fig. 3. Trajectory of the particle: (a) $c = 3.7321$; (b) $c = 3.8$; (c) $c = 3.9$; (d) $c = 3.999$.

4. Dynamic chaotic characteristics

Chaotic dynamics is defined by a deterministic system with non-regular, chaotic behavior [27]. They are both sensitive to initial conditions and computational unpredictability. The Lyapunov exponent and correlation dimension are most accessible in numerical computations based on the time-series of the dynamical system [28]. In this section, we introduce the algorithm to compute the Lyapunov exponent and correlation dimension for quantitative observation of dynamic characteristics of the particles, and then analyze the relation between chaos and the swarm intelligent model.

4.1. Lyapunov exponent

Lyapunov exponents provide a way to identify the qualitative dynamics of a system, because they describe the rate at which neighboring trajectories converge or diverge (if negative or positive, respectively) from one another in orthogonal directions. If the dynamics occur in an n-dimensional system, there are n exponents. Since the maximum exponent will dominate, this limit is practically useful only for finding the largest exponent. Chaos can be defined as the divergence between neighboring trajectories and the presence of a positive exponent could be considered as the diagnostic of chaos. For an IFS, Lyapunov exponents measure the asymptotic behavior of tangent vectors under iteration. The maximum Lyapunov exponent can be found using [29]:

$$\mathrm{Le}_1 = \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} \ln \left(\frac{d_n}{d_1} \right) \tag{8}$$

where d_n is the distance between the n th point-pair. Le_1 can be calculated using a programmable calculator to a reasonable

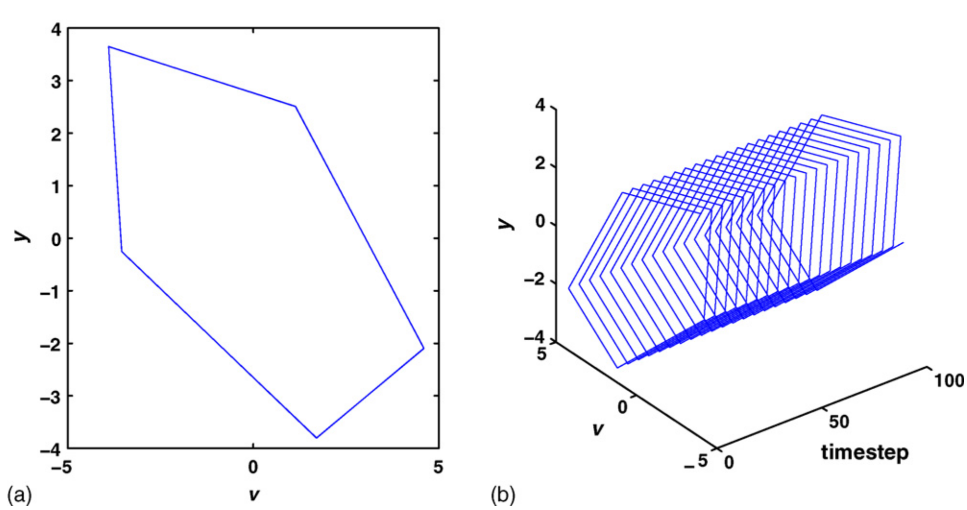

Fig. 4. 2D vs. 3D—(a) 2D: $c = 1.3820$, (b) 3D: $c = 1.3820$.

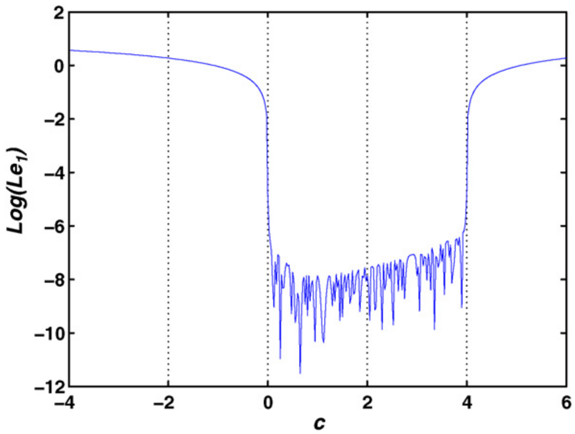

Fig. 5. Maximum Lyapunov exponent in PS.

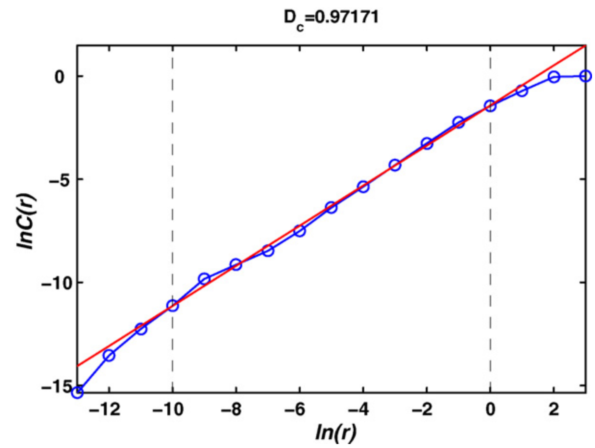

Fig. 6. Plot of $\ln C(r)$ vs. $\ln(r)$ for $c = 3.9$.

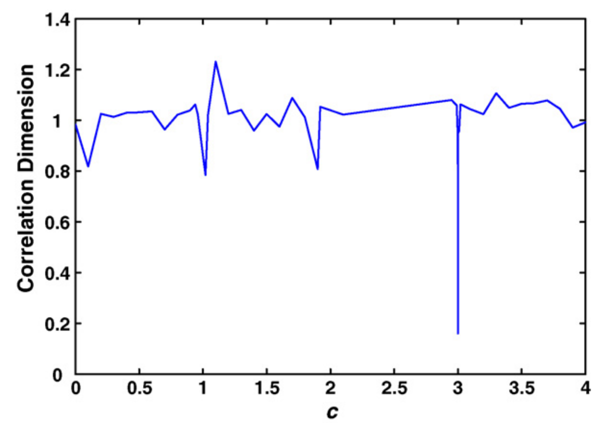

Fig. 7. Correlation dimension for varying values of c.

degree of accuracy by choosing a suitably large value of "N". Using the time-series generated from the IFS (7), the maximum Lyapunov exponent Le_1 of the particle swarm model is calculated. The results are illustrated in Fig. 5. The maximum Lyapunov exponent steadily increases with the value of c in the interval [0.5, 4] and it bounds to reach a very high level when the value of c falls within the [0, 4] interval.

4.2. Correlation dimension

The dimension in a chaotic system is a measure of its geometric scaling property or its "complexity" and it has been considered as one of the most basic properties. Numerous methods have been proposed for characterizing the dimension produced by chaotic flows and one of the most common metrics is the correlation dimension, popularized by Grassberger and Procaccia [30]. It measures the probability that two points chosen at random will be within a certain distance of each other, and examines how this probability changes as the distance is increased. During the past decades, several investigators have undertaken nonlinear analysis using Grassberger and Procaccia's algorithm (GP algorithm) to evaluate the correlation dimension of time-series data [31,32].

Given by N points $\{\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_N\}$ from the iterated processes of IFS, the definition of the correlation integral is

$$C(r) = \lim_{N\to\infty} \frac{1}{N^2} \sum_{\substack{i,j=1 \\ i\neq j}}^{N} H(r - |\vec{x}_i - \vec{x}_j|) \tag{9}$$

where $H(x)$ is the Heaviside step function. When the limit exists, the correlation dimension is then defined as (10):

$$D_2 = \lim_{r,r'\to+0} \frac{\ln(C(r)/C(r'))}{\ln(r/r')} \tag{10}$$

In practice, $C(r)$ is calculated for several values of r and then a plot is constructed for $\ln C(r)$ versus $\ln(r)$ to estimate the slope, which then approximates the correlation dimension D_2. In the particle swarm model, for $c = 3.9$, the slope, i.e. D_2 is illustrated in Fig. 6 in the interval [0, 4]. The correlation dimension is depicted in Fig. 7. There are no obvious differences for c values increasing within the interval of [0, 4]. D_2 is fluctuating mainly within 1 ± 0.2 and it is to be noted that the correlation dimension is very small when c is close to 3. Our experiment results also further validates the constant constriction coefficient w using the "classical" value $w = 1/(2\ln(2)) = 0.7213$ and the recommended value for $c = (w+1)^2 = 2.9630$ [26].

For the iterated system determined by (7), the eigenvalues of A are λ_1 and λ_2. We are looking for pair of values (c, k) so that

$$A^k = I \tag{11}$$

where I is the identity matrix. We have $\det(A) > 0$ (equal to 1, in fact), so it exists P so that

$$P^{-1}AP = \Lambda \tag{12}$$

where

$$\Lambda = [\,\lambda_1 < myamp > 0 < myslash > 0 < myamp > \lambda_2\,]$$

Eq. (11) can then be rewritten

$$(P\Lambda P^{-1})^k = \Lambda^k = I \tag{13}$$

It means we must have

$$\lambda_1^k = \lambda_2^k = 1 \tag{14}$$

But we have

$$\lambda_1 = 1 - \frac{c}{2} + \sqrt{\Delta}, \qquad \lambda_2 = 1 - \frac{c}{2} - \sqrt{\Delta} \tag{15}$$

with c is strictly positive, and

$$\Delta = \left(1 - \frac{c}{2}\right)^2 - 1$$

So it is possible for (14) only if the eigenvalues are true complex numbers, i.e. if Δ is strictly negative. It implies that c must be smaller than 4. It is easy to see that we have $|\lambda_1| = |\lambda_2| = 1$. So finally the only condition to have a perfect cycle of size k:

$$1 - \frac{c}{2} = \cos\left(\frac{2\pi}{k}\right) \tag{16}$$

i.e.:

$$c = 2\left(1 - \cos\left(\frac{2\pi}{k}\right)\right) \tag{17}$$

There are an infinity of such cycles for small c values (smaller than 1), but in [1,4] the only possible ones are for cycle sizes $k = 6, 5, 4, 3$, i.e. $c = 1, 1.382, 2, 3$. It means in particular that if we generate a sequence of points in the particle swarm model by using one of these c values, the correlation dimension will be very small. On the contrary for other values we obtain a correlation dimension near to 1, which is the value for pure random distribution.

4.3. Discussions

In order to analyze the relationship between chaos and the swarm intelligent model, we optimized three unconstrained real-valued benchmark functions, and then investigated the performance of the model against the dynamic chaotic characteristics.

First, we considered the Rastrigin's function (f_1), given by (18). It is a continuous, multi-modal function with multiple local minima. The function has a ''large scale'' curvature which guides the search towards the global minimum, $\vec{x}^* = (0, \ldots, 0)$, with $f(\vec{x}^*) = 0$ in the interval $[-5.12, 5.12]$.

Next, we considered the Zakharov's function (f_2), given by (19). It is a continuous, multi-modal function, and has the minimum, $\vec{x}^* = (0, \ldots, 0)$, with $f(\vec{x}^*) = 0$ in the interval $[-10, 10]$.

Finally, we also evaluated the Levy's function (f_3), given by (20). It is a continuous, multi-modal function with an offset, since it has the minimum, $\vec{x}^* = (1, \ldots, 1)$, with $f(\vec{x}^*) = 0$ in the interval $[-10, 10]$:

$$f_1(\vec{x}) = \sum_{i=1}^{n} [x_i^2 - 10\cos(2\pi x_i) + 10] \tag{18}$$

$$f_2(\vec{x}) = \sum_{i=1}^{n} x_i^2 + \left(\sum_{i=1}^{n} \frac{1}{2} i x_i\right)^2 + \left(\sum_{i=1}^{n} \frac{1}{2} i x_i\right)^4 \tag{19}$$

$$f_3(\vec{x}) = \frac{\pi}{n}\left(k\sin^2(\pi y_1) + \sum_{i=1}^{n-1}((y_i - a)^2(1 + k\sin^2(\pi y_{i+1}))) + (y_n - a)^2\right); \quad y_i = 1 + \frac{1}{4}(x_i - 1); \quad k = 10; \quad a = 1 \tag{20}$$

The goal of the particle swarm algorithm is to find the global minimum for functions (18) and (19). All experiments for the functions were run 10 times, and the average fitness were recorded. The swarm size was set at 20, and 1000 iterations for the trials. The results are illustrated in Fig. 8 for Rastrigin's function, Fig. 9 for Zakharov's function and Fig. 10 for Levy's function, respectively. It is obvious that the values of c within the interval [0, 4] is fit for the model because the performance is much better than the other values of c. It is consistent with the Lyapunov exponent and the correlation dimension of the model as illustrated in Figs. 5 and 7. In the interval [0, 4], the particle swarm with a high maximum Lyapunov exponent usually achieved better performance. The positive Lyapunov exponent

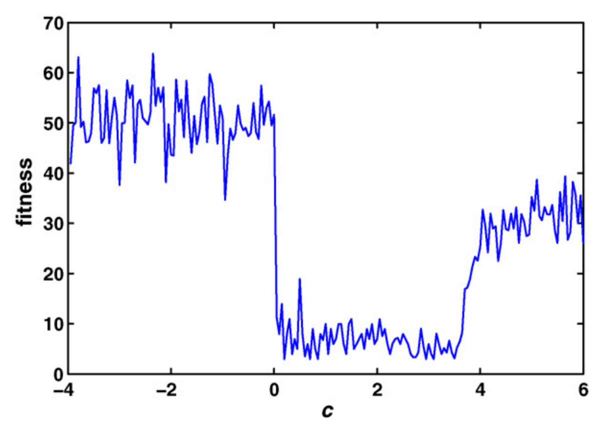

Fig. 8. The performance for 5D Rastrigin's function.

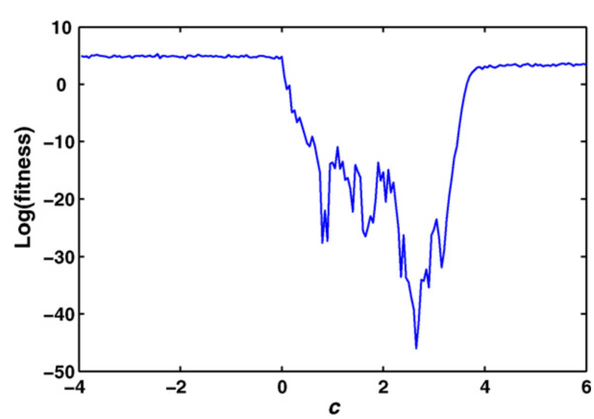

Fig. 9. The performance for 5D Zakharov's function.

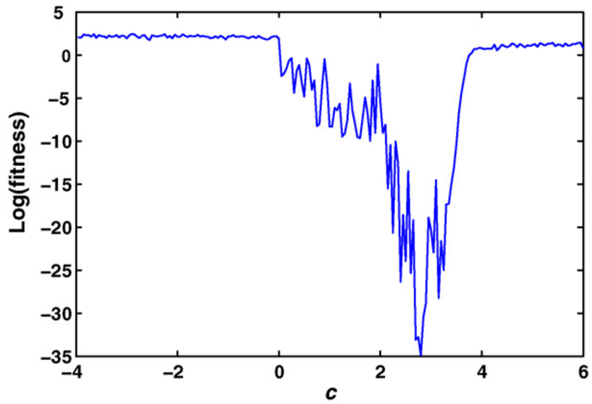

Fig. 10. The performance for 5D Levy's function.

describes the rate at which neighboring trajectories diverge. A high Lyapunov exponent in the particle swarm system implies that the particles are inclined to explore different regions and find better fitness values. But the big Lyapunov exponent would lead the system not to converge. The particles usually have to search solutions randomly because of the clamping of velocity and position. Compared to the correlation dimension of the system, the performance of the system is better when the c is close to 3.

5. Conclusions and future work

In this paper, we focused on the chaotic dynamic characteristics in swarm intelligence. Particle swarm was investigated as a case and the swarm model was represented by the iterated function system (IFS). The dynamic trajectory of the particle was sensitive on the value of the IFS parameters and the sensitivity of the system is illustrated. We introduced the algorithms to compute numerically the Lyapunov exponent and correlation dimension for quantitative observation of dynamic characteristics of the particles, and then analyzed the dependence of the parameters using some function optimization experiments. The results illustrated that the performance of the swarm intelligent model depended on the sign of the maximum Lyapunov exponent. The particle swarm with a little high maximum Lyapunov exponent usually achieved better performance, especially for the multi-modal functions. The correlation dimension of the system could recommend some values for the parameters.

Since the performance of the swarm intelligent model usually depends relatively on its Lyapunov exponent and correlation dimension, it would provide some new ideas for developing new swarm intelligent swarm models. If we can design some models with a little higher maximum Lyapunov exponent, it might be possible to construct a new swarm intelligence model with better performance. The correlation dimension of the system would provide some suggestions for the parameter selection.

Acknowledgments

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